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# Exchange and crystal field effects on the $4 \mathrm{f}^{n-1} 5 \mathrm{~d}$ levels of $\mathrm{Tb}^{3+}$ 

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#### Abstract

The levels of the $4 \mathrm{f}^{n-1} 5 \mathrm{~d}$ configuration of $\mathrm{Ce}^{3+}(n=1), \operatorname{Pr}^{3+}(n=2)$, and $\mathrm{Tb}^{3+}(n=8)$ in compounds are compared with each other. A model is presented that, by means of energy shift operations performed on the five 5 d levels of $\mathrm{Ce}^{3+}$, reproduces the energies of those for $\mathrm{Pr}^{3+}$ and $\mathrm{Tb}^{3+}$. Using $\mathrm{Tb}^{3+}$ data, two main sources of deviations from the shift model are identified. One is related to the size difference between $\mathrm{Tb}^{3+}$ and $\mathrm{Ce}^{3+}$ which affects the lattice relaxation and the crystal field splitting of the 5 d configuration. The other is related to the isotropic exchange interaction between the 5 d electron spin and the total spin of the $4 \mathrm{f}^{7}$ electrons in $\mathrm{Tb}^{3+}$. The exchange splitting is about 1 eV in fluorides, sulfates, and phosphates. In oxides with less strongly bonded oxygen 2 pelectrons, the exchange splitting decreases to 0.6 eV . The effects of the two deviations on the predictability of the $4 \mathrm{f}^{n-1} 5$ d energy levels of $\mathrm{Tb}^{3+}$ and other lanthanides are discussed.


## 1. Introduction

Evidence is accumulating rapidly that on the basis of simple energy shift operations the location of $4 \mathrm{f}^{n-1} 5 \mathrm{~d}$ energy levels of a lanthanide ion in a compound can be predicted from that observed for $\mathrm{Ce}^{3+}$ in the same compound. This is particularly evident for the first $4 \mathrm{f}^{n-1} 5 \mathrm{~d}$ energy level. The energy $E\left(n, Q\right.$, A) needed for the transition from the $4 f^{n}$ ground state to this level is given by [1]

$$
\begin{equation*}
E(n, Q, \mathrm{~A})=E_{\text {Afree }}(n, Q)-D(Q, \mathrm{~A}) . \tag{1}
\end{equation*}
$$

Here, a generalized notation is used. $n$ is the number of electrons in the $4 \mathrm{f}^{n} 5 \mathrm{~d}^{0}$ configuration, $Q=2+$ or $3+$ is the ionic charge of the lanthanide, and A stands for the name of the compound. $E_{\text {Afree }}(n, Q)$ is for each lanthanide a constant. The values for $E_{\text {Afree }}$ are close to the transition energy of the free ions. $D(n, Q, \mathrm{~A})$ is the red-shift in the lanthanide with $n$ electrons in the $4 \mathrm{f}^{n}$ configuration in compound A. It expresses the energy shift of the lowest $4 f^{n-1} 5 \mathrm{~d}$ level due to
the interaction of the 5 d electron with the crystalline environment. Within $\pm 5 \%, D(n, Q, \mathrm{~A})$ appears the same for each lanthanide. One may then omit the variable $n$, and the red-shift $D(Q, \mathrm{~A})$ becomes a property that characterizes a compound.

The red-shift for $\mathrm{Ce}^{3+}$ can be written as [2]

$$
\begin{equation*}
D(1,3+, \mathrm{A})=\epsilon_{\mathrm{c}}(1,3+, \mathrm{A})+\frac{\epsilon_{\mathrm{cfs}}(1,3+, \mathrm{A})}{r(\mathrm{~A})}-0.234 \mathrm{eV} \tag{2}
\end{equation*}
$$

where $\epsilon_{\mathrm{c}}(1,3+, \mathrm{A})$ is defined as the shift of the barycentre energy of the 5 d configuration of $\mathrm{Ce}^{3+}$ relative to the free ion value of 6.352 eV . $\epsilon_{\mathrm{cfs}}(1,3+, \mathrm{A})$ is the energy difference between the lowest and highest energy 5d states. $r(\mathrm{~A})$ depends on the shape of the anion polyhedron that coordinates $\mathrm{Ce}^{3+}$ and it determines the fraction of $\epsilon_{\mathrm{cfs}}$ that contributes to the red-shift.
$D(n, Q, \mathrm{~A})$ is almost independent of $n$ and equation (1) holds; this implies that the 5 d crystal field splitting and centroid shift are almost the same for each lanthanide ion. This was suggested before [3-5], and first experimental evidence was provided by a systematic study on the fd transitions in 11 different trivalent lanthanides in $\mathrm{CaF}_{2}, \mathrm{LiYF}_{3}$, and $\mathrm{YPO}_{4}$ [6-8]. There it was found that the centroid shift remains quite constant but the crystal field splitting decreases gradually by $10 \%$ with decreasing size of the lanthanide ion in going from $\mathrm{Ce}^{3+}$ to $\mathrm{Tm}^{3+}$. Such change in the crystal field splitting causes dispersion of $D(Q, \mathrm{~A})$ with the type of lanthanide and limits the predictive power of equation (1).

Red-shift, crystal field splitting, and centroid shift are known for $\mathrm{Ce}^{3+}$ in many compounds [2,9-12]. To explore whether the crystal field interaction changes with the type of lanthanide ion, the information on the 5d level energies of $\mathrm{Pr}^{3+}$ and $\mathrm{Tb}^{3+}$ is compared with that on $\mathrm{Ce}^{3+} . \mathrm{Pr}^{3+}$ is next to $\mathrm{Ce}^{3+}$ in the lanthanide series and has almost the same ionic radius as $\mathrm{Ce}^{3+}$ [13]. For this ion, corrections to equation (1) are not expected to be necessary. Since much spectroscopic information is available on $\mathrm{Tb}^{3+}$ and because it is $\approx 10 \mathrm{pm}$ smaller than $\mathrm{Ce}^{3+}, \mathrm{Tb}^{3+}$ is the most suitable lanthanide for use in testing whether corrections are needed for the smaller lanthanides.
$\mathrm{Tb}^{3+}$ is also of special interest because the isotropic exchange interaction between the 5 d electron spin and the total spin of the electrons in the $4 \mathrm{f}^{n-1}$ core is maximal. It leads to a difference of $\approx 1 \mathrm{eV}$ between the energy of the first spin allowed and first spin forbidden fd transition. Recently Shi and Zhang showed [14] that the exchange splitting of $\mathrm{Tb}^{3+}$ in compounds depends on the size of the nephelauxetic effect. Such change affects the red-shift $D(8,3+, \mathrm{A})$ of $\mathrm{Tb}^{3+}$ differently to the red-shift $D(1,3+, \mathrm{A})$ of $\mathrm{Ce}^{3+}$; dispersions in red-shift values are then unavoidable.

This work is organized as follows. First the theory of the $4 \mathrm{f}^{n-1} 5 \mathrm{~d}$ configuration of the lanthanides is reviewed, and different schemes of coupling between 5 d electrons and $4 \mathrm{f}^{n-1}$ electrons are considered. On the basis of the decoupled scheme, a simple model is introduced that reproduces the profile of the $4 f^{2} \rightarrow 4 f^{1} 5 d$ excitation spectrum of $\mathrm{Pr}^{3+}$. Two energy shift operations need to be performed on the known fd excitation spectrum of $\mathrm{Ce}^{3+}$.

The exchange interaction is accounted for by introducing an additional energy shift operation. It allows one to reproduce the $4 f^{8} \rightarrow 4 f^{7} 5 \mathrm{~d}$ excitation spectrum of the spin allowed and spin forbidden transitions in $\mathrm{Tb}^{3+}$ from the known spectrum of $\mathrm{Ce}^{3+}$. Data on $\mathrm{Tb}^{3+}$ are collected from existing literature, and with the predictions from the 'shift model' the $\mathrm{Tb}^{3+}$ spectra are reanalysed. It will be shown that the energy difference between the spin forbidden fd and the spin allowed fd transition decreases with the amount of covalency between the 5d orbital and anion ligands. The implications for the accuracy of equation (1) are discussed, and finally the variation of the exchange splitting with the type of lanthanide and the type of host crystal is treated.

## 2. Theory

The electronic part of the Hamiltonian describing the $4 \mathrm{f}^{n-1} 5 \mathrm{~d}^{1}$ configuration is written as [15-17]

$$
\begin{equation*}
H=H_{\mathrm{ff}}^{\mathrm{C}}+H_{\mathrm{ff}}^{\mathrm{ex}}+H_{\mathrm{fd}}^{\mathrm{C}}+H_{\mathrm{fd}}^{\mathrm{ex}}+V_{\mathrm{f}}+V_{\mathrm{d}}+H_{\mathrm{f}}^{\mathrm{so}}+H_{\mathrm{d}}^{\mathrm{so}} \tag{3}
\end{equation*}
$$

where $H_{\mathrm{ff}}^{\mathrm{C}}$ and $H_{\mathrm{ff}}^{\mathrm{ex}}$ are the Coulomb and exchange interactions between the electrons in the $4 \mathrm{f}^{n-1}$ configuration. $H_{\mathrm{fd}}^{\mathrm{C}}$ and $H_{\mathrm{fd}}^{\mathrm{ex}}$ are the same interactions between 5 d electrons and $4 \mathrm{f}^{n-1}$ electrons. $V_{\mathrm{f}}$ and $V_{\mathrm{d}}$ describe the interaction of 4 f electrons and 5 d electrons with the crystal field. $H_{\mathrm{f}}^{\text {so }}$, and $H_{\mathrm{d}}^{\text {so }}$ are the spin-orbit interactions of the f and d electrons.

In the case of $\mathrm{Ce}^{3+}$ all terms involving f electrons are absent, and then the crystal field splitting $\epsilon_{\mathrm{cfs}}$ and the centroid shift $\epsilon_{\mathrm{c}}$ are determined by $V_{\mathrm{d}}$ and

$$
\begin{equation*}
H_{\mathrm{d}}^{\mathrm{so}}=\zeta_{\mathrm{d}} \overrightarrow{\mathrm{l}}_{\mathrm{d}} \cdot \vec{s}_{\mathrm{d}} \tag{4}
\end{equation*}
$$

where $\zeta_{\mathrm{d}}$ is the 5 d spin-orbit interaction parameter. In compounds with sites of low symmetry, the effect of the 5 d spin-orbit interaction is quenched by the crystal field splitting, and then $V_{\mathrm{d}}$ is the only remaining interaction.

For the other lanthanides one may safely ignore $V_{\mathrm{f}}$ because the interaction between 4 f electrons and the crystal field is very small [16]. If also the interactions between 5 d and $4 \mathrm{f}^{n-1}$ electrons is ignored, we arrive at the decoupled scheme

$$
\begin{equation*}
H_{0}=H_{\mathrm{ff}}^{\mathrm{C}}+H_{\mathrm{ff}}^{\mathrm{ex}}+H_{\mathrm{f}}^{\mathrm{so}}+V_{\mathrm{d}} . \tag{5}
\end{equation*}
$$

The levels of $4 \mathrm{f}^{n-1} 5 \mathrm{~d}^{1}$ are written as $4 \mathrm{f}^{n-1}\left[{ }^{2 S+1} \mathrm{~L}_{J}\right] 5 \mathrm{~d}_{i}$, i.e., as a product of the eigenstates of the $4 \mathrm{f}^{n-1}$ configuration with the eigenstates of $5 \mathrm{~d}^{1}$. The ${ }^{2 S+1} \mathrm{~L}_{J}$ levels are determined by the first three terms in equation (5). The energy of $5 \mathrm{~d}_{i}$, where the index $i=1,2, \ldots, 5$ denotes the five levels in order of increasing energy, are determined by $V_{\mathrm{d}}$.

The isotropic exchange interaction between the 5 d electron spin $\vec{s}_{\mathrm{d}}$ and the total spin $\vec{S}_{\mathrm{f}}$ of the $4 \mathrm{f}^{n-1}$ electrons is given by [15]

$$
\begin{equation*}
H_{\mathrm{fd}}^{\mathrm{ex}}=-2 J_{0} \vec{S}_{\mathrm{d}} \cdot \vec{S}_{\mathrm{f}} \tag{6}
\end{equation*}
$$

with $J_{0}$ as the exchange interaction strength. In the case of high values for $\vec{S}_{\mathrm{f}}$, like in $\mathrm{Eu}^{3+}, \mathrm{Gd}^{3+}$, $\mathrm{Tb}^{3+}, \mathrm{Sm}^{2+}, \mathrm{Eu}^{2+}$, and $\mathrm{Gd}^{2+}, H_{\mathrm{fd}}^{\mathrm{ex}}$ is stronger than $H_{\mathrm{f}}^{\text {so }}$. Ignoring $H_{\mathrm{f}}^{\text {so }}$, the levels of $4 \mathrm{f}^{n-1} 5 \mathrm{~d}^{1}$ may be written as $4 \mathrm{f}^{n-1}\left[{ }^{2 S_{\mathrm{f}}+1} \mathrm{~L}_{\mathrm{f}}\right] 5 \mathrm{~d}_{i}[\mathrm{HS}]$ and $4 \mathrm{f}^{n-1}\left[{ }^{2 S_{\mathrm{f}}+1} \mathrm{~L}_{\mathrm{f}}\right] 5 \mathrm{~d}_{i}[\mathrm{LS}]$ where the attachment [HS] and [LS] indicates whether the 5 d electron spin is parallel (high spin) or anti-parallel (low spin) to $\vec{S}_{\mathrm{f}}$.

Figure 1 shows the energy level scheme of the $4 f^{7}\left[{ }^{8} \mathrm{~S}\right] 5 \mathrm{~d}$ configuration of free $\mathrm{Gd}^{2+}$ and free $\mathrm{Tb}^{3+}[18,19]$. The average energy of the levels is chosen as the zero of energy. The most important interaction is $H_{\mathrm{fd}}^{\text {ex }}$ causing a separation of $8 J_{0}$ between the high spin $[\mathrm{HS}]^{9} \mathrm{D}_{J}$ and the low spin $[\mathrm{LS}]^{7} \mathrm{D}_{J}$ multiplets. The spin-orbit coupling causes the additional Landé interval splitting of ${ }^{9} \mathrm{D}_{J}$ and ${ }^{7} \mathrm{D}_{J}$. The exchange and spin-orbit splitting are larger in $\mathrm{Tb}^{3+}$ which is related to the 18 pm smaller size of $\mathrm{Tb}^{3+}$.

In compounds the orbital angular momentum of the 5 d electron is quenched and then $H_{\mathrm{f}}^{\text {so }}$ commutes with the total angular momentum [15, 16]:

$$
\begin{equation*}
J^{\prime}=S_{\mathrm{f}}+s_{\mathrm{d}}+L_{\mathrm{f}} \equiv S^{\prime}+L_{\mathrm{f}} \tag{7}
\end{equation*}
$$

with

$$
\begin{equation*}
H_{\mathrm{f}}^{\mathrm{so}}=\zeta \vec{S}^{\prime} \cdot \vec{L}_{\mathrm{f}} \tag{8}
\end{equation*}
$$

In this $\left(S^{\prime}, L_{\mathrm{f}}\right) J^{\prime}$ coupling scheme, the energy levels are written as $4 \mathrm{f}^{n-1}\left[{ }^{2 S_{\mathrm{f}}+1} \mathrm{~L}_{\mathrm{f}}\right] 5 \mathrm{~d}_{i}[\mathrm{HS}]_{J^{\prime}}$ and $4 \mathrm{f}^{n-1}\left[{ }^{2 S_{\mathrm{f}}+1} \mathrm{~L}_{\mathrm{f}}\right] 5 \mathrm{~d}_{i}[\mathrm{LS}]_{J^{\prime}}$. For $\mathrm{Tb}^{3+}$ and $\mathrm{Gd}^{2+}$ a particularly simple situation is obtained because $L_{\mathrm{f}}=0$ and the Landé splitting of the ${ }^{9} \mathrm{D}$ and ${ }^{7} \mathrm{D}$ levels vanishes.


Figure 1. The energy level scheme showing the effect of the exchange splitting and spin-orbit splitting on the $4 f^{7}\left[{ }^{8} \mathrm{~S}\right] 5 \mathrm{~d}$ states of free $\mathrm{Gd}^{2+}$ and free $\mathrm{Tb}^{3+}$.
$\left(S^{\prime}, L_{\mathrm{f}}\right) J^{\prime}$ coupling can be used for the lanthanides up to $\mathrm{Tb}^{3+}$ and $\mathrm{Gd}^{2+}$. Proceeding further to the end of the lanthanide series, the exchange interaction decreases but the spinorbit interaction becomes much stronger. For lanthanides such as $\mathrm{Tm}, \mathrm{Yb}, \mathrm{Lu}$, the $\left(J_{\mathrm{f}} j_{\mathrm{d}}\right) J^{\prime}$ coupling scheme is more appropriate [20,21]. First the spin and angular momentum of the $4 \mathrm{f}^{n-1}$ core couple to $J_{\mathrm{f}}$ and those of the 5 d electron couple to $j_{\mathrm{d}}$. The $J_{\mathrm{f}}$ and $j_{\mathrm{d}}$ coupling is performed next leading to further substructure in the level scheme.

## 3. The shift model

In this section a model for generating the $4 \mathrm{f}^{n} \rightarrow 4 \mathrm{f}^{n-1} 5 \mathrm{~d}$ excitation or absorption spectra of trivalent lanthanides is presented. It is a practical model meant to roughly predict or interpret fd excitation spectra without entering into too much theoretical detail. Information on the energy of each 5 d level of $\mathrm{Ce}^{3+}$ and on the $4 \mathrm{f}^{n-1}$ level energies of the lanthanide is needed as input. By means of energy shift operations performed on these 5 d energies the model, hereafter called the 'shift model', constructs the $4 \mathrm{f}^{n-1} 5 \mathrm{~d}$ energy level diagram for other lanthanides. One element of the shift model is already expressed by the red-shift $D(Q, \mathrm{~A})$ in equation (1). With information on the energy of the lowest 5 d level of $\mathrm{Ce}^{3+}$ in a compound, the first $4 \mathrm{f}^{n-1} 5 \mathrm{~d}$ level of other lanthanides can be predicted by applying the red-shift $D(Q, \mathrm{~A})$.

One may choose the decoupled, the $\left(S^{\prime}, L_{\mathrm{f}}\right) J^{\prime}$, or the $\left(J_{\mathrm{f}}, j_{\mathrm{d}}\right) J^{\prime}$ coupling schemes as the basis for the shift model. Figure 2 illustrates the shift model applied to $\mathrm{Pr}^{3+}$ in $\mathrm{LiYF}_{4}$ using the decoupled scheme. The energies of the five $5 \mathrm{~d}_{i}(i=1, \ldots, 5)$ levels of $\mathrm{Ce}^{3+}$ in $\mathrm{LiYF}_{4}$ are shown in the left part of figure 2. In the case of $\mathrm{Pr}^{3+}$ one may simply shift the levels by $E_{\text {Afree }}(2,3+)-E_{\text {Afree }}(1,3+)=1.52 \mathrm{eV}$ to obtain the 4 f 5 d levels with the 4 f electron occupying the ${ }^{2} \mathrm{~F}_{5 / 2}$ ground state level. The crystal field splitting between the 5 d levels is maintained because $V_{\mathrm{d}}$ for $\mathrm{Pr}^{3+}$ is assumed the same as for $\mathrm{Ce}^{3+}$. We will denote this shift operation as $\mathrm{Sh}_{\mathrm{d}}$.

The electron remaining in $4 f^{1}$ can also be found in the ${ }^{2} \mathrm{~F}_{7 / 2}$ state leading to an additional set of five 4 f 5 d levels at higher energy. This additional shift denoted as $\mathrm{Sh}_{\mathrm{f}}$ is the same as


Figure 2. Experimental 5d level energies of $\mathrm{Ce}^{3+}$ in $\mathrm{LiYF}_{4}$ and the level positions for $\mathrm{Pr}^{3+}, \mathrm{Tb}^{3+}$, and $\mathrm{Er}^{3+}$ generated by the shift model.
the difference between the ${ }^{2} \mathrm{~F}_{5 / 2}$ and the ${ }^{2} \mathrm{~F}_{7 / 2}$ level of the $4 \mathrm{f}^{1}$ core. It is 0.19 and 0.26 eV in the $4 f^{1}$ configuration of $\mathrm{La}^{2+}$ and $\mathrm{Ce}^{3+}$. For the $4 \mathrm{f}^{1} 5 \mathrm{~d}$ configuration of $\mathrm{Pr}^{3+}$ we estimate a slightly larger spin-orbit splitting of $\approx 0.3 \mathrm{eV}$ because of the contraction of the $4 \mathrm{f}^{1}$ core after fd excitation.

As an example, figure 3 shows the excitation spectra of df emission of $\mathrm{Ce}^{3+}$ and $\mathrm{Pr}^{3+}$ in $\mathrm{NaMgF}{ }_{3}$ [22]. The five bands of $\mathrm{Ce}^{3+}$ are nicely resolved. The shift operations $\mathrm{Sh}_{\mathrm{d}}$ and $\mathrm{Sh}_{\mathrm{f}}$, illustrated by the set of vertical bars above the spectra, reproduce the main features, i.e., the position but also intensity of the $\mathrm{Pr}^{3+}$ excitation spectrum. The same was observed for $\mathrm{Ce}^{3+}$ and $\mathrm{Pr}^{3+}$ in $\mathrm{KMgF}_{3}$ [22], and in $\mathrm{CaSO}_{4}, \mathrm{SrSO}_{4}$, and $\mathrm{BaSO}_{4}$ [25], and in $\mathrm{LaPO}_{4}$ [24].

The appropriate coupling scheme for $\mathrm{Pr}^{3+}$ is $\left(S^{\prime}, L_{\mathrm{f}}\right) J^{\prime}$ coupling resulting in three separate levels ( $J^{\prime}=4,3,2$ ) for each $5 \mathrm{~d}_{i}$ state instead of two. Usually, as in figure 3 , the corresponding excitation bands are not resolved because of strong electron-phonon coupling and because of overlap with higher $5 d_{i}$ states. Exceptions are provided by $\mathrm{CaF}_{2}, \mathrm{LiYF}_{4}$, and $\mathrm{YPO}_{4}$ where the electron-phonon coupling is very weak and the energy separation between the $5 \mathrm{~d}_{1}$ and the next higher non-degenerate $5 \mathrm{~d}_{i}$ state is larger than the splitting between $J^{\prime}$ states. Van Pieterson et al $[6,7]$ found for these compounds two subbands at 0.17 and 0.45 eV higher energy than the first 4 f 5 d band. In our notation the states can be written as $4 \mathrm{f}^{1}\left[{ }^{2} \mathrm{~F}\right] 5 \mathrm{~d}_{1}[\mathrm{HS}]_{4,3,2}$.

In those cases where $\left(J_{\mathrm{f}}, j_{\mathrm{d}}\right)$ coupling is more appropriate, each ${ }^{2 S+1} \mathrm{~L}_{J}$ level of $4 \mathrm{f}^{n-1}$ defines a $\mathrm{Sh}_{\mathrm{f}}$ operation. The $\mathrm{Sh}_{\mathrm{f}}$ energies can be estimated from the Dieke diagram of the trivalent lanthanides [23]. For example, the fd excitation spectrum of $\mathrm{Er}^{3+}$ in $\mathrm{LiYF}_{4}$ is well reproduced by $\mathrm{Sh}_{\mathrm{d}}=E_{\text {Afree }}(11,3+)-E_{\text {Afree }}(1,3+)=3.78 \mathrm{eV}$ together with $\mathrm{Sh}_{\mathrm{f}}=0,0.65$, and 1.08 eV ; see figure 2. The $\mathrm{Sh}_{\mathrm{f}}$ values were taken as the same as the energy differences between the ${ }^{5} \mathrm{I}_{8},{ }^{5} \mathrm{I}_{7}$, and ${ }^{5} \mathrm{I}_{6}$ states of the $4 \mathrm{f}^{10}$ configuration in free $\mathrm{Ho}^{3+}$ [23]. Within 0.15 eV the first five fd bands, denoted as A to F in figure 2, correspond with experiment [7].


Figure 3. The $4 \mathrm{f} \rightarrow 5 \mathrm{~d}$ and $4 \mathrm{f}^{2} \rightarrow 4 \mathrm{f}^{1} 5 \mathrm{~d}$ excitation spectra of $\mathrm{Ce}^{3+}$ and $\mathrm{Pr}^{3+}$ luminescence in $\mathrm{NaMgF}_{3}$. The vertical bars above the $\mathrm{Pr}^{3+}$ spectrum indicate $\operatorname{Pr} 5 \mathrm{~d}$ level energies generated by the shift model from the $\mathrm{Ce}^{3+} 5 \mathrm{~d}$ energies.
(This figure is in colour only in the electronic version)

The lanthanide of special interest in this work is $\mathrm{Tb}^{3+}$. With $\mathrm{Sh}_{\mathrm{d}}=E_{\text {Afree }}(8,3+)-$ $E_{\text {Afree }}(1,3+)=1.64 \mathrm{eV}$ the five main levels of the $4 \mathrm{f}^{7} 5 \mathrm{~d}$ configuration are obtained; see figure 2. The levels with the $4 f^{7}$ core in the ${ }^{6} \mathrm{P}$ state can be generated with $\mathrm{Sh}_{\mathrm{f}}=4.07 \mathrm{eV}$, i.e., the same energy difference as between ${ }^{8} \mathrm{~S}$ and ${ }^{6} \mathrm{P}$ of the $4 \mathrm{f}^{7}$ configuration in $\mathrm{Gd}^{3+}$. Since this shift is always larger than the total crystal field splitting of the 5 d levels, the $4 \mathrm{f}^{7}\left[{ }^{6} \mathrm{P}\right] 5 \mathrm{~d}_{i}$ states are well separated from the five $4 f^{7}\left[{ }^{8} \mathrm{~S}\right] 5 \mathrm{~d}_{i}$ states.

The effect of the isotropic exchange interaction can be accounted for by introducing a third type of shift operation. Performing $\mathrm{Sh}_{\mathrm{ex}}=-1 \mathrm{eV}$ on the $\left.4 \mathrm{f}^{7}{ }^{8}{ }^{8} \mathrm{~S}\right] 5 \mathrm{~d}_{i}[\mathrm{LS}]$ states yields the $4 \mathrm{f}^{7}\left[{ }^{8} \mathrm{~S}\right] 5 \mathrm{~d}_{i}[\mathrm{HS}]$ states. We can do the same with the $4 \mathrm{f}^{7}\left[{ }^{6} \mathrm{P}\right] 5 \mathrm{~d}_{i}[\mathrm{LS}]$ states. Because of the smaller $\vec{S}_{\mathrm{f}}$ the splitting between [HS] and [LS] is $6 J_{0}$ instead of $8 J_{0}$, and then $\mathrm{Sh}_{\mathrm{ex}} \approx-0.75 \mathrm{eV}$. Note that the transition to $4 \mathrm{f}^{7}\left[{ }^{6} \mathrm{P}\right] 5 \mathrm{~d}_{1}[\mathrm{HS}]$ anticipated at 9.23 eV is the spin allowed one. The so-called $J$ and $J^{\prime}$ bands observed by van Pieterson et al $[26]$ in $\mathrm{LiYF}_{4}: \mathrm{Tb}^{3+}$ at 9.47 and 9.32 eV are most probably related to this transition.

## 4. Results and discussion

There are two main points of interest in this work. (1) To what extent can the energy of the $4 \mathrm{f}^{n-1} 5 \mathrm{~d}_{i}$ levels be predicted by applying the shift model to the known fd excitation spectrum of $\mathrm{Ce}^{3+}$ ? (2) What are the causes for dispersion of $D(n, 3+, \mathrm{A})$ with $n$ and how do they influence the predictive power of equation (1). We will treat two probable causes for dispersion.
(1) The ionic radius of the lanthanide ion decreases by 16 pm in going from $\mathrm{La}^{3+}$ to $\mathrm{Lu}^{3+}$ [13]. Unavoidably the lattice relaxation changes with the size of the lanthanide, and this affects the interaction $V_{\mathrm{d}}$.
(2) The charge cloud expansion of the 5d orbital (nephelauxetic effect) and mixing with ligand orbitals (covalency) reduce the isotropic exchange interaction.

Such an effect was recently noticed for several $\mathrm{Tb}^{3+}$ doped compounds [14]. It is also a known phenomenon for the $\mathrm{s}^{2}$ elements $\mathrm{Tl}^{+}, \mathrm{Pb}^{2+}, \mathrm{Bi}^{3+}$ where the exchange splitting between ${ }^{3} \mathrm{P}$ and ${ }^{1} \mathrm{P}$ terms of the sp excited configuration is strongly reduced in compounds [27].

We will focus on comparing the experimental fd excitation spectra of $\mathrm{Pr}^{3+}$ and $\mathrm{Tb}^{3+}$ with that of $\mathrm{Ce}^{3+}$. Because of the relatively low energy of the 5 d levels these are the most widely studied trivalent lanthanides. Furthermore, the spectroscopy of the $4 \mathrm{f}^{n-1} 5 \mathrm{~d}$ configuration is relatively simple for these lanthanides.

Data on $\mathrm{Pr}^{3+}$ and especially $\mathrm{Tb}^{3+}$ were critically (re)analysed using the shift model. Identification of the first two intense spin allowed and the first weak spin forbidden fd transition in $\mathrm{Tb}^{3+}$ is usually not problematic. However, there are the following uncertainties on how to assign the higher energy levels of the $4 f^{7} 5 d$ configuration:
(1) Figure 2 illustrates that when the energy difference between $5 \mathrm{~d}_{1}$ and $5 \mathrm{~d}_{2}$ is smaller than 0.75 eV , one may observe below the first $5 \mathrm{~d}_{1}[\mathrm{LS}]$ state the spin forbidden transition to $5 \mathrm{~d}_{1}[\mathrm{HS}]$ and $5 \mathrm{~d}_{2}[\mathrm{HS}]$ states.
(2) One may not assume beforehand that the orbital angular momentum of the 5d electron is fully quenched in compounds. Although, this was suggested in several theoretical treatments [15-17], it has not been verified for $\mathrm{Tb}^{3+}$. If the assumption does not hold, then the ${ }^{9} \mathrm{D}$ state splits further and more than one spin forbidden transition could be observed below the first spin allowed one.
(3) The exchange interaction $J_{0}$ may depend on the type of compound. The interaction may also be different for the higher $5 \mathrm{~d}_{i}$ states.
(4) It is not known what intensities to expect for the spin forbidden transitions as compared to the spin allowed ones.
To analyse $\mathrm{Tb}^{3+}$ fd excitation spectra, the shift model combined with information on $\mathrm{Ce}^{3+}$ is used to assign the different bands observed, and vice versa the bands observed are used to verify to what extent the shift model holds. To test and illustrate the shift model, a spectroscopic study was done on $\mathrm{Ce}^{3+}$ and $\mathrm{Tb}^{3+}$ doped $\mathrm{LiLuSiO}_{4}$. Figure 4 shows the excitation spectra of $\mathrm{Ce}^{3+} \mathrm{df}$ emission at 400 nm and $\mathrm{Tb}^{3+}$ ff emission at 545 nm . The experimental techniques used can be found elsewhere [22]. The first three $\mathrm{Ce}^{3+} 5 \mathrm{~d}$ excitation bands are found at 3.51, 3.91 , and 4.08 eV . The other two, anticipated around 6.4 eV , are too weak to be detected. The fundamental absorption due to excitation of the $\mathrm{SiO}_{4}^{4-}$ groups starts at $E_{\mathrm{fa}}=7.15 \mathrm{eV}$.

On lowering the temperature to 10 K (spectrum 2), the $5 \mathrm{~d}_{2}$ and $5 \mathrm{~d}_{3}$ bands narrow and can be resolved in the spectrum. Application of the shift operation $\mathrm{Sh}_{\mathrm{d}}=1.69 \mathrm{eV}$ to the first three $\mathrm{Ce}^{3+}$ levels provides the $\mathrm{Tb}^{3+} 4 \mathrm{f}^{7}\left[{ }^{8} \mathrm{~S}\right] 5 \mathrm{~d}_{i}[\mathrm{LS}]$ level energies. They agree with intense excitation bands observed at 5.23 and 5.64 eV . Note that the splitting between $5 \mathrm{~d}_{2}$ and $5 \mathrm{~d}_{3}$ is 0.023 eV smaller than for $\mathrm{Ce}^{3+}$ and the two bands are not resolved in spectrum (3). Probably the asymmetry on the high energy side of the 5.64 eV band indicates the presence of the $5 \mathrm{~d}_{3}$ band. The 50 and 10 times weaker bands at 4.31 and 4.75 eV are attributed to the $5 \mathrm{~d}_{1}[\mathrm{HS}]$ and the unresolved $5 \mathrm{~d}_{2,3}[\mathrm{HS}]$ bands of $\mathrm{Tb}^{3+}$. A shift operation $\mathrm{Sh}_{\mathrm{ex}}=-0.92 \mathrm{eV}$ on the [LS] $\mathrm{Tb}^{3+}$ bands reproduces nicely the energies of the observed high spin bands. Spectrum (4) measured with a higher wavelength resolution and longer photon counting time reveals a narrow $\mathrm{Tb}^{3+} 4 \mathrm{f}^{8} \rightarrow 4 \mathrm{f}^{8}$ transition at 3.89 eV . The spin forbidden fd excitation bands are much wider and do not reveal any substructure.

Based on the $\mathrm{LiLuSiO}_{4}$ data and data to be presented further on in this work, the following five conclusions are drawn:
(1) $\mathrm{The} \mathrm{Sh}_{\mathrm{d}}$ shift operation provides the main bands of $\mathrm{Tb}^{3+}$.
(2) The exchange splitting between [HS] and [LS] bands is the same for each $5 \mathrm{~d}_{i}$, and the [HS] levels can be found by using the $\mathrm{Sh}_{\mathrm{ex}}$ shift operation.


Figure 4. Excitation spectra of $400 \mathrm{~nm} \mathrm{Ce}{ }^{3+}$ df emission in $\mathrm{LiLuSiO}_{4}$ at 295 K (spectrum (1)) and 10 K (spectrum (2)). Spectrum (3) is the excitation spectrum of $545 \mathrm{~nm} \mathrm{~Tb}^{3+}$ ff emission in $\mathrm{LiLuSiO}_{4}$ at 10 K . Na salicylate was used to correct for the light source spectral profile. Spectrum (4) is like spectrum (3) but with longer counting time and smaller wavelength step size. The wavelength resolution is 0.3 nm . Vertical bars illustrate band positions generated by the shift model.
(3) The intensity of the transition to the $5 \mathrm{~d}_{1}[\mathrm{HS}]$ level is $10-50$ times weaker than that to the $5 \mathrm{~d}_{1}[\mathrm{LS}]$ level.
(4) The intensity of the transition to the $5 \mathrm{~d}_{2}[\mathrm{HS}]$ level is $5-10$ times stronger than that to $5 \mathrm{~d}_{1}[\mathrm{HS}]$.
(5) Only one single spin forbidden transition to $5 \mathrm{~d}_{1}[\mathrm{HS}]$ is observed. The absence of a further splitting of $5 \mathrm{~d}_{1}[\mathrm{HS}]$ confirms that the 5 d angular momentum is quenched in compounds and that the $H_{\mathrm{f}}^{\text {so }}$ interaction, because $L_{\mathrm{f}}=0$, can be ignored for $\mathrm{Tb}^{3+}$.

### 4.1. Presentation of data

A large part of the information on fd transitions in $\mathrm{Ce}^{3+}, \mathrm{Pr}^{3+}$, and $\mathrm{Tb}^{3+}$ together with references can be found in previous work [2, 9-12, 28]. Since the appearance of that work, more information has been retrieved from the literature. Not all data is presented; instead a subset of the most relevant data are tabulated. Table 1 for example compiles data on those compounds where both the $5 \mathrm{~d}_{1}[\mathrm{HS}]$ and $5 \mathrm{~d}_{1}[\mathrm{LS}]$ levels of $\mathrm{Tb}^{3+}$ were identified.

Figure 5 shows the energies of the first allowed fd absorption and df emission in $\mathrm{Ce}^{3+}$, $\mathrm{Pr}^{3+}$, and $\mathrm{Tb}^{3+}$ against that in $\mathrm{Ce}^{3+}$. For this figure, the full collection of available data was used. When equation (1) holds, all data must fall on sets of parallel lines with unit slope. It applies best to $\mathrm{Pr}^{3+}$ where the energy difference between the fd transition in $\mathrm{Pr}^{3+}$ and that in $\mathrm{Ce}^{3+}$ averaged over $N=110$ data points amounts to $1.51 \pm 0.09 \mathrm{eV}$. Note that the $\pm 3 \mathrm{~nm}$ uncertainty in the absorption and emission wavelengths of $\mathrm{Ce}^{3+}$ and $\mathrm{Pr}^{3+}$ already accounts for the $\pm 0.08 \mathrm{eV}$ error in the energy difference. Therefore, within experimental accuracy, equation (1) applies and $D(2,3+, \mathrm{A})=D(1,3+, \mathrm{A})$.

The allowed first fd transition in $\mathrm{Tb}^{3+}$ is on average $(N=75)$ at $1.66 \pm 0.12 \mathrm{eV}$ higher energy than that in $\mathrm{Ce}^{3+}$. Like for $\mathrm{Pr}^{3+}$ the experimental uncertainty is $\pm 0.08 \mathrm{eV}$. In the following sections two causes for the additional dispersion of 0.08 eV leading to the overall dispersion of 0.12 eV are considered. (1) $V_{\mathrm{d}}$ for $\mathrm{Tb}^{3+}$ is different to that for $\mathrm{Ce}^{3+}$ leading to

Table 1. The energies $(\mathrm{eV})$ of the first spin allowed $E^{\text {sa }}(8,3+, \mathrm{A})$ and the first spin forbidden $E^{\text {sf }}(8,3+, \mathrm{A})$ transitions of $\mathrm{Tb}^{3+}$ in compounds A. The intensity ratios $I^{\text {sf }} / I^{\text {sa }}$ of the spin forbidden to spin allowed transitions in excitation spectra are specified, together with the $\mathrm{Tb}^{3+}$ concentration within brackets. $\Delta E_{1}^{\mathrm{ex}}=E^{\mathrm{sa}}-E^{\text {sf }}$. All energies are in electron volts.

| Compound | $E^{\text {sf }}$ | $E^{\text {sa }}$ | $\Delta E_{1}^{\text {ex }}$ | $I^{\text {sf }} / I^{\text {sa }}$ | References |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{CaF}_{2}$ | 4.79 | 5.77 | 0.980 |  | [7, 37, 38] |
| $\mathrm{LiYF}_{4}$ | 4.86 | 5.88 | 1.014 | 0.017 (1\%) | [7, 39, 40] |
| Acetonitrile ( $\left.\mathrm{RECl}_{6}\right)^{3-}$ | 4.56 | 5.30 | 0.740 | 0.02 (100\%) | [41, 42] |
| Acetonitrile ( $\left.\mathrm{REBr}_{6}\right)^{3-}$ | 4.46 | 5.12 | 0.663 |  | [41, 42] |
| $\alpha$-GdOF | 4.77 | 5.51 | 0.742 | 0.07 (2\%) | [43] |
| $\beta$-GdOF | 4.40 | 5.17 | 0.769 | 0.06 (2\%) | [43] |
| YOF | 4.31 | 5.17 | 0.861 | 0.03 (5\%) | [44] |
| $\mathrm{Y}_{3}\left(\mathrm{SiO}_{4}\right)_{2} \mathrm{Cl}$ | 4.53 | 5.32 | 0.796 | 0.1 (4.3\%) | [45] |
| LuOCl | 3.88 | 4.75 | 0.876 | 0.027 (2\%) | [46, 47] |
| ScOCl | 4.13 | 4.88 | 0.749 | 0.014 (2\%) | [46] |
| LaOBr | 4.29 | 4.86 | 0.572 | 0.1 (7.5\%) | [48-50] |
| YOBr | 4.26 | 4.86 | 0.602 | 0.04 (5\%) | [44] |
| LaOI | 3.82 | 4.70 | 0.882 | 0.01 (0.6\%) | [47, 51] |
| $\mathrm{CaSO}_{4}$ | 4.86 | 5.82 | 0.959 | 0.023 (2\%) | [52] |
| $\mathrm{Y}_{2} \mathrm{O}_{2}\left(\mathrm{SO}_{4}\right)$ | 4.51 | 5.39 | 0.882 |  | [53] |
| $\mathrm{TbP}_{5} \mathrm{O}_{14}$ | 4.82 | 5.71 | 0.889 | 0.32 (100\%) | [54, 55] |
| $\mathrm{LaP}_{3} \mathrm{O}_{9}$ | 4.91 | 5.88 | 0.966 | 0.05 (5\%) | [56] |
| $\mathrm{GdP}_{3} \mathrm{O}_{9}$ | 4.90 | 5.85 | 0.948 | 0.03 (5\%) | [56] |
| $\mathrm{KTbP}_{4} \mathrm{O}_{12}$ | 4.77 | 5.74 | 0.971 | 0.33 (100\%) | [55] |
| $\mathrm{YP}_{3} \mathrm{O}_{9}$ | 4.73 | 5.71 | 0.981 | 0.03 (5\%) | [56] |
| $\mathrm{LaPO}_{4}$ | 5.12 | 6.05 | 0.925 | 0.05 (4\%) | [24, 56, 57] |
| $\mathrm{K}_{3} \mathrm{La}\left(\mathrm{PO}_{4}\right)_{2}$ | 4.88 | 5.69 | 0.806 | 0.5 (20\%) | [58, 59] |
| $\mathrm{GdPO}_{4}$ | 5.00 | 6.02 | 1.019 | 0.04 (5\%) | [56, 60] |
| $\mathrm{TbPO}_{4}$ | 4.77 | 5.64 | 0.867 | 0.44 (100\%) | [61] |
| $\mathrm{K}_{3} \mathrm{~Tb}\left(\mathrm{PO}_{4}\right)_{2}$ | 4.70 | 5.61 | 0.914 | 0.3 (100\%) | [ $55,59,62]$ |
| $\beta-\mathrm{Na}_{3} \mathrm{Gd}\left(\mathrm{PO}_{4}\right)_{2}$ | 4.71 | 5.49 | 0.772 | 0.08 (0.5\%) | [63] |
| $\mathrm{YPO}_{4}$ | 4.66 | 5.56 | 0.899 | 0.07 (0.11\%) | [7, 56, 64] |
| $\mathrm{LuPO}_{4}$ | 4.70 | 5.54 | 0.839 |  | [64] |
| $\mathrm{ScPO}_{4}$ | 4.56 | 5.44 | 0.880 |  | [64] |
| Aqueous $\left[\mathrm{Tb}\left(\mathrm{OH}_{2}\right)_{8}\right]^{3+}$ | 4.71 | 5.66 | 0.947 |  | [65-67] |
| $\mathrm{BaLaB} 9 \mathrm{O}_{16}$ | 5.00 | 5.99 | 0.990 | 0.05 (10\%) | [55, 68] |
| $\mathrm{BaGdB}_{9} \mathrm{O}_{16}$ | 5.06 | 5.99 | 0.929 | 0.06 (20\%) | [68, 69] |
| $\mathrm{LaB}_{3} \mathrm{O}_{6}$ | 5.39 | 6.20 | 0.809 | 0.016 (5\%) | [56, 70] |
| $\mathrm{Ca}_{3}\left(\mathrm{BO}_{3}\right)_{2}$ | 4.40 | 5.19 | 0.791 | 0.02 (0.3\%) | [71, 72] |
| $\mathrm{LaBO}_{3}$ | 4.61 | 5.49 | 0.877 | 0.033 (20\%) | [44, 56] |
| $\mathrm{GdAl}_{3}\left(\mathrm{BO}_{3}\right)_{4}$ | 4.71 | 5.61 | 0.896 |  | [73] |
| $\mathrm{TbAl}_{3}\left(\mathrm{BO}_{3}\right)_{4}$ | 4.63 | 5.59 | 0.959 | 0.17 (100\%) | [74, 75] |
| $\mathrm{GdBO}_{3}$ | 4.41 | 5.28 | 0.864 | 0.15 (20\%) | [44, 56, 76, 78] |
| $\mathrm{TbBO}_{3}$ | 4.44 | 5.28 | 0.832 | 0.30 (100\%) | [61] |
| $\mathrm{Na}_{6} \mathrm{Gd}\left(\mathrm{BO}_{3}\right)_{3}$ | 4.08 | 4.81 | 0.727 | 0.085 | [79] |
| $\mathrm{YBO}_{3}$ | 4.38 | 5.25 | 0.873 | 0.08 (20\%) | [ $56,78,80]$ |
| Calcite $\mathrm{LuBO}_{3}$ | 4.40 | 5.34 | 0.948 |  | [81] |
| $\mathrm{ScBO}_{3}$ | 4.29 | 5.19 | 0.898 | 0.06 (0.13\%) | [44, 76, 78, 81] |
| $\mathrm{InBO}_{3}$ | 4.40 | 5.28 | 0.879 | 0.05 (0.11\%) | [77, 81] |
| $\beta-\mathrm{Y}_{2} \mathrm{Si}_{2} \mathrm{O}_{7}$ | 4.35 | 5.23 | 0.881 | 0.074 | [82, 83] |
| $\mathrm{Mg}_{2} \mathrm{SiO}_{4}$ | 4.16 | 5.08 | 0.921 | 0.07 (9\%) | [55, 84] |
| $\mathrm{Gd}_{3} \mathrm{Mg}_{2} \mathrm{GaGe}_{2} \mathrm{O}_{12}$ | 3.51 | 4.31 | 0.793 | 0.03 | [85] |
| $\mathrm{Y}_{3} \mathrm{Mg}_{2} \mathrm{GaGe}_{2} \mathrm{O}_{12}$ | 3.47 | 4.28 | 0.802 | (1\%) | [85] |
| $\mathrm{LiLuSiO}_{4}$ | 4.29 | 5.21 | 0.919 | 0.019 | This work |

Table 1. (Continued.)

| Compound | $E^{\text {sf }}$ | $E^{\text {sa }}$ | $\Delta E_{1}^{\text {ex }}$ | $I^{\text {sf }} / I^{\text {sa }}$ | References |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $\mathrm{X} 1-\mathrm{Gd}_{2} \mathrm{SiO}_{5}:(\mathrm{Ce} 2)$ | 4.01 | 4.96 | 0.947 | $0.083(2 \%)$ | $[86]$ |
| $\mathrm{X} 1-\mathrm{Y}_{2} \mathrm{SiO}_{5}$ | 4.32 | 5.08 | 0.761 | $0.07(0.12 \%)$ | $[87,88]$ |
| $\mathrm{X}_{2}-\mathrm{Y}_{2} \mathrm{SiO}_{5}:(\mathrm{Ce} 1)$ | 4.29 | 5.15 | 0.855 | 0.09 | $[83,88-90]$ |
| $\mathrm{Y}_{2} \mathrm{Ca}_{2}\left(\mathrm{Si}_{2} \mathrm{O}_{7}\right) \mathrm{O}_{2}$ | 4.35 | 5.28 | 0.926 | $0.05(5 \%)$ | $[44]$ |
| $\mathrm{Mg}_{2} \mathrm{Y}_{8}\left(\mathrm{SiO}_{4}\right)_{6} \mathrm{O}_{2}:(6 \mathrm{~h})$ | 4.51 | 5.23 | 0.723 | $0.06(2.7 \%)$ | $[91]$ |
| $\mathrm{CaAl}_{2} \mathrm{O}_{4}: \mathrm{Ce}_{2}$ | 4.38 | 5.04 | 0.659 | $0.49(1 \%)$ | $[92,93]$ |
| $\mathrm{SrLaGa}_{3} \mathrm{O}_{7}$ | 4.38 | 5.17 | 0.785 | $0.19(80 \%)$ | $[94]$ |
| $\mathrm{GdAlO}_{3}$ | 4.90 | 5.66 | 0.761 | $0.20(2 \%)$ | $[95,96]$ |
| $\mathrm{Gd}_{3} \mathrm{Ga}_{5} \mathrm{O}_{12}$ | 3.95 | 4.63 | 0.678 | $0.31(6 \%)$ | $[97]$ |
| $\mathrm{Y}_{3} \mathrm{Al}_{5} \mathrm{O}_{12}$ | 3.86 | 4.54 | 0.679 | $(0.03)(1 \%)$ | $[44,98,99]$ |
| $\mathrm{Y}_{3} \mathrm{Al}_{4} \mathrm{GaO}_{12}$ | 3.88 | 4.63 | 0.752 |  | $[98]$ |
| $\mathrm{Y}_{3} \mathrm{Al}_{3} \mathrm{Ga}_{2} \mathrm{O}_{12}$ | 3.91 | 4.64 | 0.732 | 0.09 | $[98]$ |
| $\mathrm{Y}_{3} \mathrm{Al}_{2} \mathrm{Ga}_{3} \mathrm{O}_{12}$ | 3.97 | 4.68 | 0.705 | - | $[44,98]$ |
| $\mathrm{Y}_{3} \mathrm{AlGa}_{4} \mathrm{O}_{12}$ | 3.99 | 4.68 | 0.692 | 0.11 | $[98]$ |
| $\mathrm{Y}_{3} \mathrm{Ga}_{5} \mathrm{O}_{12}$ | 4.00 | 4.70 | 0.697 | $0.24(5 \%)$ | $[44,98,100]$ |
| $\mathrm{La}_{2} \mathrm{Hf}_{2} \mathrm{O}_{7}$ | 4.43 | 5.10 | 0.674 | $0.10(1 \%)$ | $[101]$ |
| $\mathrm{SrLa}_{2} \mathrm{BeO}_{5}$ | 4.07 | 4.77 | 0.704 | $0.16(1 \%)$ | $[102]$ |
| $\mathrm{LiSr}_{2} \mathrm{YO}_{4}$ | 3.35 | 3.88 | 0.524 | $0.15(10 \%)$ | $[103]$ |
| $\mathrm{C}_{4} \mathrm{Lu}_{2} \mathrm{O}_{3}$ | 3.54 | 4.05 | 0.509 | $0.004(1 \%)$ | $[104]$ |



Figure 5. $E(n, 3+, \mathrm{A})$ for $\mathrm{Ce}^{3+}, \mathrm{Pr}^{3+}$, and $\mathrm{Tb}^{3+}$ against $E(1,3+, \mathrm{A})$ for $\mathrm{Ce}^{3+}$. Open symbols are data on $\mathrm{f} \rightarrow \mathrm{d}$ absorption transitions and solid symbols data on $\mathrm{d} \rightarrow \mathrm{f}$ emission transitions. For presentation purposes, the data for $\mathrm{Tb}^{3+}$ are offset by 1.0 eV . Dashed lines have unit slope.
different crystal field splitting and centroid shift. (2) The exchange interaction $H_{\mathrm{fd}}^{\mathrm{ex}}$ depends on the type of compound.

The shift model works for the first allowed fd transition in $\mathrm{Tb}^{3+}$. To test the model for the transitions to higher 5d levels and the spin forbidden ones, data were collected on the energies of these transitions also. Table 2 compiles the energies of the five $5 \mathrm{~d}_{i}$ levels of $\mathrm{Ce}^{3+}$. With the shift model applied to these data, $4 \mathrm{f}^{8} \rightarrow 4 \mathrm{f}^{7} 5 \mathrm{~d}$ excitation spectra of $\mathrm{Tb}^{3+}$ in compounds were interpreted. The energies of the $4 \mathrm{f}^{7}\left[{ }^{8} \mathrm{~S}\right] 5 \mathrm{~d}_{i}[\mathrm{LS}]$ and $4 \mathrm{f}^{7}\left[{ }^{8} \mathrm{~S}\right] 5 \mathrm{~d}_{i}[\mathrm{HS}]$ identified are compiled

Table 2. Energies $E_{i}(i=1, \ldots, 5)$ of the $5 \mathrm{~d}_{i}[\mathrm{HS}]$ and $5 \mathrm{~d}_{i}[\mathrm{LS}]$ states of $\mathrm{Ce}^{3+}$ and $\mathrm{Tb}^{3+}$ in compounds in electron volts.

| Compound | Ln | $E_{5}$ | $E_{4}$ | $E_{3}$ | $E_{2}$ | $E_{1}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{LaF}_{3}$ | Ce | 6.39 | 5.96 | 5.69 | 5.30 | 4.98 |
|  | Tb [LS] | 8.09 | 7.64 | 7.24 | 6.87 | 6.47 |
|  | Tb [HS] | 7.06 | 6.64 | 6.17 | - | - |
| $\mathrm{YF}_{3}$ | Ce | 6.39 | 6.11 | 5.74 | 5.19 | 4.84 |
|  | Tb [LS] | 8.05 | 7.75 | 7.47 | 6.91 | 6.46 |
|  | Tb [HS] | 7.07 | 6.69 | 6.29 | 5.82 | - |
| $\mathrm{LiYF}_{4}$ | Ce | 6.67 | 6.33 | 6.02 | 5.08 | 4.25 |
|  | Tb [LS] | 8.21 | 8.05 | 7.75 | 6.81 | 5.88 |
|  | Tb [HS] | - | - | - | - | 4.86 |
| $\mathrm{LaP}_{3} \mathrm{O}_{9}$ | Ce | 6.39 | 6.05 | 5.41 | 4.68 | 4.28 |
|  | Tb [LS] | - | - | 7.07 | 6.25 | 5.88 |
|  | Tb [HS] | - | 6.72 | 5.96 | 5.22 | 4.91 |
| $\mathrm{LaPO}_{4}$ | Ce | 6.02 | 5.79 | 5.19 | 4.84 | 4.54 |
|  | Tb [LS] | - | - | 6.88 | 6.36 | 6.05 |
|  | Tb [HS] | 6.70 | 6.52 | 5.82 | 5.49 | 5.12 |
| $\mathrm{YPO}_{4}$ | Ce | 6.11 | 5.51 | 5.21 | 4.96 | 3.85 |
|  | Tb [LS] | 7.70 | 7.17 | 6.93 | 6.74 | 5.56 |
|  | Tb [HS] | - | 6.20 | 5.93 | 5.71 | 4.66 |
| $\mathrm{LaB}_{3} \mathrm{O}_{6}$ | Ce | 6.08 | 5.66 | 5.04 | 4.77 | 4.59 |
|  | Tb [LS] | - | 7.40 | 6.67 | 6.39 | 6.20 |
|  | Tb [HS] | - | - | - | 5.66 | 5.39 |
| $\mathrm{LaMgB}_{5} \mathrm{O}_{10}$ | Ce | 6.14 | 5.51 | 5.19 | 4.82 | 4.56 |
|  | Tb [LS] | - | 7.17 | 6.89 | 6.39 | 6.08 |
|  | Tb [HS] | - | - | - | - | - |
| $\mathrm{LaBO}_{3}$ | Ce | 5.77 | 5.34 | 5.15 | 4.66 | 3.76 |
|  | Tb [LS] | - | - | 6.70 | 6.33 | 5.49 |
|  | Tb [HS] | - | 6.11 | 5.82 | - | 4.61 |
| $\mathrm{YBO}_{3}$ | Ce | 6.20 | 5.66 | 5.06 | 3.67 | 3.47 |
|  | Tb [LS] | - | - | 6.74 | 5.44 | 5.25 |
|  | Tb [HS] | - | - | 5.90 | 4.58 | 4.38 |
| Calcite $\mathrm{LuBO}_{3}$ | Ce | 6.81 | 6.42 | 4.09 | 3.82 | 3.65 |
|  | Tb [LS] | - | - | - | - | 5.34 |
|  | Tb [HS] | - | - | 4.77 | 4.56 | 4.40 |
| $\mathrm{ScBO}_{3}$ | Ce | 6.74 | 6.46 | 3.88 | 3.62 | 3.46 |
|  | Tb [LS] | - | - | - | - | 5.19 |
|  | Tb [HS] | - | - | 4.71 | 4.46 | 4.29 |
| $\mathrm{Mg}_{2} \mathrm{SiO}_{4}$ | Ce | - | - | - | 3.85 | 3.32 |
|  | Tb [LS] | - | - | 6.36 | 5.56 | 5.08 |
|  | Tb [HS] | - | - | - | 4.68 | 4.16 |
| $\mathrm{LiLuSiO}_{4}$ | Ce | - | - | 4.16 | 3.91 | 3.56 |
|  | Tb [LS] | - | - | 5.71 | 5.61 | 5.21 |
|  | Tb [HS] | - | - | 4.77 | 4.71 | 4.29 |
| $\mathrm{GdAlO}_{3}$ | Ce | 5.39 | 5.06 | 4.48 | 4.32 | 4.04 |
|  | Tb [LS] | - | - | - | 5.98 | 5.66 |
|  | Tb [HS] | - | - | - | 5.23 | 4.90 |
| $\mathrm{Y}_{3} \mathrm{Al}_{5} \mathrm{O}_{12}$ | Ce | 6.05 | 5.51 | 4.75 | 3.65 | 2.71 |
|  | Tb [LS] | - | - | - | 5.45 | 4.54 |
|  | Tb [HS] | - | - | 5.99 | - | 3.86 |



Figure 6. The difference between the $\mathrm{Ce}^{3+} 5 \mathrm{~d}$ red-shift $D(1,3+, \mathrm{A})$ and the $\mathrm{Tb}^{3+} 5 \mathrm{~d}$ red-shift $D(8,3+, \mathrm{A})$ as a function of the crystal field splitting contribution, $\epsilon_{\mathrm{cfs}}(1,3+, \mathrm{A}) / r(\mathrm{~A})$, to the $\mathrm{Ce}^{3+}$ red-shift.
in table 2. The literature on the $\mathrm{Ce}^{3+}$ data can be found in $[2,10-12]$ and references therein. The $\mathrm{Tb}^{3+}$ data on $\mathrm{LaF}_{3}$ and $\mathrm{YF}_{3}$ are from [29,30] and those on $\mathrm{LaMgB}_{5} \mathrm{O}_{10}$ are from [31]. For other $\mathrm{Tb}^{3+}$ data the same sources as cited in table 1 were used.

### 4.2. Deviations from the shift model due to $V_{d}$

Crystal field splitting is caused by exchange (Pauling repulsion) and Coulomb interaction between 5 d electron and anion ligands. Suppose $\mathrm{Ce}^{3+}$ is replaced by the smaller $\mathrm{Tb}^{3+}$ ion and lattice relaxation does not occur. The Coulomb and exchange interaction is less and consequently $V_{\mathrm{d}}$ and the crystal field splitting decrease. Although lattice relaxation partly cancels this effect, smaller crystal field splitting is still expected for $\mathrm{Tb}^{3+}$. For $\mathrm{CaF}_{2}, \mathrm{YPO}_{4}$, and $\mathrm{LiYF}_{4}$ van Pieterson et al $[6,7]$ found that the 5 d crystal field splitting of $\mathrm{Tb}^{3+}$ is $5 \%$ smaller than that of $\mathrm{Ce}^{3+}$. A $9 \%$ decrease was observed for the even smaller lanthanides $\mathrm{Tm}^{3+}$ and $\mathrm{Yb}^{3+}$.

A crystal field splitting that depends on the size of the lanthanide ion affects the red-shift $D(3+, \mathrm{A})$; see equation (2). $D(n, 3+, \mathrm{A})$ then depends slightly on $n$ and dispersion of the data in figure 5 is introduced. Figure 6 shows the difference between the red-shifts in $\mathrm{Ce}^{3+}$ and in $\mathrm{Tb}^{3+}$ against $\epsilon_{\mathrm{cfs}}(1,3+, \mathrm{A}) / r(\mathrm{~A})$. A clear correlation exists. For compounds with large crystal field splitting, like in $\mathrm{Y}_{3} \mathrm{Al}_{5} \mathrm{O}_{12}$ with the garnet structure, $D(1,3+, \mathrm{A})-D(8,3+, \mathrm{A})>0$.

For compounds providing a large lattice site for $\mathrm{Tb}^{3+}$ another effect may occur. When the anion coordination number is large $(\geqslant 8)$ and the crystal field splitting small, lattice relaxation distorts the shape of the anion coordination polyhedron which may lead to augmented crystal field splitting. In that case, $D(1,3+, \mathrm{A})-D(8,3+$, A) can be smaller than zero. This is the situation in $\mathrm{LaF}_{3}$. The assignment of bands in table 2 indicates that the $\mathrm{Tb}^{3+}$ crystal field splitting is $13 \%$ larger in $\mathrm{LaF}_{3}$ than the splitting for $\mathrm{Ce}^{3+}$. The variation of the crystal field splitting with the size of the lanthanide is now regarded as a main contribution to the dispersion from equation (1).


Figure 7. The relation between the exchange splitting $\Delta E_{1}^{\mathrm{ex}}$ and the energy $E^{\mathrm{sa}}(8,3+, \mathrm{A})$ of the $4 f^{7}\left[{ }^{8} S\right] 5 \mathrm{~d}_{1}[\mathrm{LS}]$ level in $\mathrm{Tb}^{3+}$.

### 4.3. The deviations from the shift model due to $H_{f d}^{e x}$

The exchange splitting $\Delta E_{1}^{\text {ex }}$, defined as the energy difference between $5 \mathrm{~d}_{1}[\mathrm{LS}]$ and $5 \mathrm{~d}_{1}[\mathrm{HS}]$, is shown in figure 7 against the energy $E^{\text {sa }}(8,3+, A)$ of the first spin allowed transition in $\mathrm{Tb}^{3+}$. One observes a clear decrease with smaller $E^{\text {sa }}(8,3+, \mathrm{A})$. The data extrapolate to the value of $8 J_{0}=1.27 \mathrm{eV}$ of the free ion. Further analysis reveals that $\Delta E_{1}^{\mathrm{ex}}$ does not correlate with the 5 d crystal field splitting, but it does correlate with the size of the centroid shift. Figure 8 shows $\Delta E_{1}^{\text {ex }}$ against the centroid shift of the $\mathrm{Ce}^{3+} 5 \mathrm{~d}$ configuration which can be calculated from the $\mathrm{Ce}^{3+}$ data in table 2. An almost linear relationship is observed that again extrapolates to the value $8 J_{0}=1.27 \mathrm{eV}$ for free $\mathrm{Tb}^{3+}$.

Another way to demonstrate a relation between the exchange splitting and type of compound is via the so-called spectroscopic polarizability $\alpha_{\mathrm{sp}}$ of anions. This parameter is a measure for the amount of covalency between 5 d and anion ligands and for the correlated motion of 5d electrons with ligand electrons which are the two most important contributions to the centroid shift of the 5 d configuration [2]. It was recently found that $\alpha_{\mathrm{sp}}$ has a very simple relationship with the mean value $\chi_{\mathrm{av}}$ of the electronegativity of the cations in the compound [32]:

$$
\begin{equation*}
\alpha_{\mathrm{sp}}=\alpha_{0}+\frac{b}{\chi_{\mathrm{av}}^{2}} \tag{9}
\end{equation*}
$$

where $\alpha_{0}$ is $0.33 \times 10^{-30} \mathrm{~m}^{3}$ and $0.15 \times 10^{-30} \mathrm{~m}^{3}$ for oxygen and fluorine, respectively. $b$ is the susceptibility of the anion to change in its polarizability by binding to cations. It is $4.8 \times 10^{-30} \mathrm{~m}^{3}$ and $0.96 \times 10^{-30} \mathrm{~m}^{3}$ for oxygen and fluorine, respectively. $\chi_{\mathrm{av}}$ is defined as [32]

$$
\begin{equation*}
\chi_{\mathrm{av}}=\frac{1}{\gamma N_{\mathrm{a}}} \sum_{i}^{N_{\mathrm{c}}} z_{i} \chi_{i} \tag{10}
\end{equation*}
$$



Figure 8. The exchange splitting $\Delta E_{1}^{\mathrm{ex}}$ against the centroid shift $\epsilon_{\mathrm{c}}(1,3+, \mathrm{A})$ of the 5 d configuration of $\mathrm{Ce}^{3+}$ in compounds.
where $N_{\mathrm{a}}$ is the number of anions in the compound formula. The summation is over all $N_{\mathrm{c}}$ cations where $-\gamma$ and $z_{i}$ are the charges of the anion and the cation $i . \chi_{i}$ is the electronegativity of cation $i$ as compiled by Allred [33].

Figure 9 shows the exchange splitting in $\mathrm{Tb}^{3+}$ in fluoride and oxide compounds against $\alpha_{\text {sp }}$ calculated with equations (9) and (10). This much larger collection of data follows the same pattern as in figure 8. Shi and Zhang [14] recently related the exchange splitting observed for $\mathrm{Tb}^{3+}$ in eight different compounds to Jörgensen's nephelauxetic $h_{\mathrm{e}}$ parameter [34], and found a linear relationship:

$$
\begin{equation*}
\Delta E_{\text {ex }}(8,3+, \mathrm{A})=1.09-0.37 h_{\mathrm{e}} \mathrm{eV} \tag{11}
\end{equation*}
$$

where the parameter $h_{\mathrm{e}}$ was calculated from the covalency between Tb and anions and from the polarizability of the bonds [14]. Relation (11) is very similar to that in figure 9. This is not too surprising since $\alpha_{\text {sp }}$ and $h_{\mathrm{e}}$ are related parameters and more or less proportional to each other [35].

Because the size of the exchange splitting depends on the type of compound whereas in $\mathrm{Ce}^{3+}$ the exchange splitting is absent, red-shift values $D(n, 3+, \mathrm{A})$ cannot be constant with $n$ and dispersion is unavoidable. Figure 10 shows the energy of the transition to $5 \mathrm{~d}_{1}[\mathrm{HS}]$ and $5 d_{1}[\mathrm{LS}]$ of $\mathrm{Tb}^{3+}$. The reduction of exchange splitting is seen as a less than unit slope of the line through the $5 \mathrm{~d}_{1}[\mathrm{HS}]$ data. Fortunately, the data on $5 \mathrm{~d}_{1}[\mathrm{LS}]$ still follow a line of unit slope. It means that the reduction of the exchange splitting does not affect the red-shift of the $5 \mathrm{~d}_{1}[\mathrm{LS}]$ level. However, it does reduce the red-shift of $5 \mathrm{~d}_{1}[\mathrm{HS}]$. Note that the data on the [HS] and [LS] $5 \mathrm{~d}_{1}$ levels of $\mathrm{Tb}^{3+}$ extrapolate to the barycentre of the ${ }^{9} \mathrm{D}_{J}(6.588 \mathrm{eV})$ and ${ }^{7} \mathrm{D}_{J}(7.857 \mathrm{eV})$ states of free $\mathrm{Tb}^{3+}$. This provides additional evidence for the quenching of the orbital momentum of the 5 d electron due to the large 5 d crystal field splitting in compounds. It also evidences that the $\left(S^{\prime}, L_{\mathrm{f}}\right) J^{\prime}$ coupling scheme applies for $\mathrm{Tb}^{3+}$.

Summarizing, it is found that equation (1) with $E_{\text {Afree }}(n, 3+)=6.118,7.625$, and 7.780 eV applies to $\mathrm{Ce}^{3+}, \mathrm{Pr}^{3+}$, and $\mathrm{Tb}^{3+}$, respectively. Within the experimental accuracy of $\pm 0.08 \mathrm{eV}, D(1,3+, \mathrm{A})=D(2,3+, \mathrm{A})$. The main reason that $D(8,3+, \mathrm{A})$ may deviate


Figure 9. The exchange splitting $\Delta E_{1}^{\mathrm{ex}}$ against the calculated spectroscopic polarizability $\alpha_{\text {sp }}$ of the anions in the compound.


Figure 10. The energy of the first spin allowed and the first spin forbidden transition in $\mathrm{Tb}^{3+}$ against the energy of the first fd transition in $\mathrm{Ce}^{3+}$. The dashed line through the [LS] data has unit slope, and that through the [HS] data has slope 0.85 .
slightly but significantly from $D(1,3+, \mathrm{A})$ is the smaller size of $\mathrm{Tb}^{3+}$. The lattice relaxation is different to that for $\mathrm{Ce}^{3+}$, and this influences the crystal field interaction $V_{\mathrm{d}}$. The reduction of $H_{\mathrm{fd}}^{\text {ex }}$ in compounds mainly affects the energy of the $5 \mathrm{~d}_{i}[\mathrm{HS}]$ state in $\mathrm{Tb}^{3+}$ and not that of the $5 \mathrm{~d}_{i}[\mathrm{LS}]$ state.

The reduction of $H_{\mathrm{fd}}^{\mathrm{ex}}$ is attributed to mixing of the 5 d orbital with anion ligand orbitals. It reduces the spin purity of the state and therefore the size of the exchange splitting in equation (6).

One then expects also an increase of the oscillator strength of the 'spin forbidden' transitions relative to that of the 'spin allowed' ones. To test this, the intensity ratio $I^{\text {sf }} / I^{\text {sa }}$ between the $5 \mathrm{~d}_{1}[\mathrm{HS}]$ and $5 \mathrm{~d}_{1}[\mathrm{LS}]$ bands was determined and the data are compiled in table 1 . Conclusive evidence is difficult to obtain because the oscillator strength also depends on other properties such as the site symmetry. Furthermore, the ratio $I^{\text {sf }} / I^{\text {sa }}$ depends on the $\mathrm{Tb}^{3+}$ concentration. Nevertheless, $I^{\text {sf }} / I^{\text {sa }}$ is about 0.02 in fluorides and in phosphates with strongly bonded anion electrons. In silicates and aluminates where bonding is weaker, the ratio tends to increase to values between 0.05 and 0.10 .

What appears very clearly from the data is an increase in the oscillator strength for the transitions to the higher $5 \mathrm{~d}_{i}[\mathrm{HS}]$ states as compared to transitions to $5 \mathrm{~d}_{1}[\mathrm{HS}]$. This can already be seen in figure 4 where the $5 \mathrm{~d}_{2}[\mathrm{HS}]$ transition is ten times stronger than the $5 \mathrm{~d}_{1}[\mathrm{HS}]$ transition. The same is generally observed in other compounds also. For the even higher $5 \mathrm{~d}_{i}$ states, the spin forbidden transitions are often of comparable intensity to the spin allowed ones. Only by using the shift model and information available on $\mathrm{Ce}^{3+}$ can a distinction between the spin forbidden and spin allowed bands then be made.

### 4.4. Generalization to other lanthanides

The shift model and equation (1) work very well for $\mathrm{Pr}^{3+}$ and with somewhat larger error bars also for the smaller $\mathrm{Tb}^{3+}$ ion. For the even smaller lanthanides such as $\mathrm{Tm}^{3+}$ and $\mathrm{Lu}^{3+}$ the error is likely to increase further.
$\vec{S}_{\mathrm{f}}$ reduces from $7 / 2$ for $\mathrm{Tb}^{3+}$ to $1 / 2$ for $\mathrm{Yb}^{3+}$. Assuming that $J_{0}(n, 3+, \mathrm{A})$ does not depend on $n$ and A, the splitting between the $5 \mathrm{~d}_{1}[\mathrm{HS}]$ and $5 \mathrm{~d}_{1}[\mathrm{LS}]$ states then decreases from $8 J_{0}$ to $2 J_{0}$. However, $J_{0}$ is not a constant. Figure 11 shows the estimated $J_{0}(n, 2+$, free $)$ values for the free divalent lanthanides. The estimates were made by means of a parabolic fit through $J_{0}(n, 2+$, free $)$ values reported for $\mathrm{Ce}^{2+}(0.21 \mathrm{eV}), \operatorname{Pr}^{2+}(0.20 \mathrm{eV}), \mathrm{Tm}^{2+}(0.17 \mathrm{eV})$, and $\mathrm{Yb}^{2+}(0.164 \mathrm{eV})$ in Yanase [16] and values for $\mathrm{Eu}^{2+}(0.135 \mathrm{eV})$ and $\mathrm{Gd}^{2+}(0.133 \mathrm{eV})$ from Spector and Sugar [19] and Callahan [18]. Since $J_{0}\left(8,3+\right.$, free) in $\mathrm{Tb}^{3+}$ is $19 \%$ larger than $J_{0}(8,2+$, free $)$ in $\mathrm{Gd}^{2+}$-see figure $1-$ we assume for each lanthanide $J_{0}(n, 3+$, free $)=$ $1.19 J_{0}(n, 2+$, free).

Figure 11 shows the energies $E_{\mathrm{HS}}^{\mathrm{ex}}$ and $E_{\mathrm{LS}}^{\mathrm{ex}}$ calculated with equation (6) for trivalent lanthanides. $E_{\mathrm{HS}}^{\mathrm{ex}}$ is the energy by which the high spin states are lowered, and $E_{\mathrm{LS}}^{\mathrm{ex}}$ is the energy by which the low spin states are raised due to the exchange interaction; see figure 1. For free ions, starting from $\mathrm{Tb}^{3+}$ with $n=8$ and going up to $\mathrm{Lu}^{3+}$ with $n=14$, the total exchange splitting decreases from $8 J_{0}=1.27$ to $2 J_{0}=0.41 \mathrm{eV}$. In compounds, $H_{\mathrm{fd}}^{\mathrm{ex}}$ is further reduced by at least $25 \%$; see figure 9 . One then expects the exchange splitting to decrease from 0.95 eV for $\mathrm{Tb}^{3+}$ towards 0.31 for $\mathrm{Lu}^{3+}$. Such a trend is indeed observed experimentally $[7,28,36]$. However, the situation with the spin forbidden transitions in the heavy lanthanides is more complicated than that in $\mathrm{Tb}^{3+}$. $L_{\mathrm{f}}$ is non-zero and this leads within the $\left(S^{\prime}, L_{\mathrm{f}}\right) J^{\prime}$ coupling scheme to more then one $5 \mathrm{~d}_{i}[\mathrm{HS}]$ level. In Dy ${ }^{3+}$, for example, two $5 \mathrm{~d}_{1}[\mathrm{HS}]$ states are observed below the $5 \mathrm{~d}_{1}[\mathrm{LS}]$ state [7].

## 5. Summary and conclusions

fd excitation spectra of $\mathrm{Ce}^{3+}, \mathrm{Pr}^{3+}$, and $\mathrm{Tb}^{3+}$ luminescence in compounds were gathered and reanalysed. Some of the data obtained on the energy of the $4 \mathrm{f}^{n-1} 5 \mathrm{~d}$ levels are compiled in tables 1 and 2. A method has been introduced that generates the energies of the main $4 \mathrm{f}^{n-1} 5 \mathrm{~d}$ levels in $\mathrm{Pr}^{3+}$ and $\mathrm{Tb}^{3+}$ in a compound from the known energies of the five 5 d levels of $\mathrm{Ce}^{3+}$ in that same compound. The method is based on the assumption that the crystal field splitting


Figure 11. The exchange interaction parameter $J_{0}(n, Q, \mathrm{~A})$ for the $\bigcirc$ trivalent $(Q=3+)$ and $\diamond$ divalent $(Q=2+)$ free lanthanide ions $(\mathrm{A}=$ free $) . \nabla$ : the exchange interaction energy $\left(E_{\mathrm{LS}}^{\mathrm{ex}}\right)$ of the low spin states; $\nabla$ : that ( $E_{\mathrm{HS}}^{\mathrm{ex}}$ ) for the high spin states.
and centroid shift of the 5 d levels, i.e., the interaction $V_{\mathrm{d}}$ in equation (3), are the same for each lanthanide ion. An upward shift by an amount $\mathrm{Sh}_{\mathrm{d}}$ of the energy of each of the five 5 d levels provides the energies for $\mathrm{Pr}^{3+}$ or $\mathrm{Tb}^{3+}$; see figures 3 and 4. $\mathrm{Sh}_{\mathrm{d}}$ is for each lanthanide a different constant that does not depend on the type of compound. By introducing a set of $\mathrm{Sh}_{\mathrm{f}}$ energy shift operations, the levels belonging to states with excited $4 \mathrm{f}^{n-1}$ cores can be generated. The effect of the isotropic exchange interaction between the 5 d electron spin and total spin of the $4 f^{n-1}$ electrons can be accounted for by a third shift operation $\left(\mathrm{Sh}_{\mathrm{ex}}\right)$.

The so-called shift model is a crude model with limitations. It does not account for the $H_{\mathrm{f}}^{\text {so }}$ interaction—see equation (8)—responsible for additional structure in the $\operatorname{Pr}^{3+} 4 \mathrm{f} 5 \mathrm{~d}$ level scheme. The situation is much more favourable for $\mathrm{Tb}^{3+}$. Because the 5 d spin-orbit interaction is quenched in compounds and because $L_{\mathrm{f}}=0$ for $\mathrm{Tb}^{3+}$, in theory the interaction $H_{\mathrm{f}}^{\text {so }}=0$. Furthermore, the $4 f^{7}\left[{ }^{6} \mathrm{P}\right] 5 \mathrm{~d}_{1}[\mathrm{HS}]$ state is always higher than the $4 \mathrm{f}^{7}\left[{ }^{7} \mathrm{~S}\right] 5 \mathrm{~d}_{5}[\mathrm{LS}]$ state. Because of this, the shift model applies particularly well for $\mathrm{Tb}^{3+}$. This makes $\mathrm{Tb}^{3+}$ ideally suited for testing to what extent $V_{\mathrm{d}}$ is a lanthanide invariant and $H_{\mathrm{fd}}^{\mathrm{ex}}$ a compound invariant interaction.

The following conclusions were drawn:
(1) The 10 pm smaller size of $\mathrm{Tb}^{3+}$ as compared to $\mathrm{Ce}^{3+}$ results in different crystal field splittings. They tend to be smaller for $\mathrm{Tb}^{3+}$ when a small site is occupied. When large sites are occupied, a lattice relaxation may result in an enhanced splitting. The lanthanide contraction and its effect on crystal field splitting are regarded as the main contributions to the dispersion in the red-shift values $D(n, Q, \mathrm{~A})$ with $n$; see equation (1) and figure 6 .
(2) The exchange splitting between [HS] and [LS] states is in a first approximation the same for each $5 \mathrm{~d}_{i}$ state.
(3) The size of the exchange splitting decreases with increasing covalency between 5 d orbital and ligand orbitals. This results in an almost linear decrease with increasing centroid shift-see figure 8-or with increasing anion polarizability-see figure 9. At the same time, the oscillator strength of the spin forbidden transition to $5 \mathrm{~d}_{1}[\mathrm{HS}]$ tends to increase;
see table 1. Both effects were explained by the decrease in the spin purity of the $5 \mathrm{~d}_{1}$ state due to orbital mixing.
(4) The intensity of transitions to $5 \mathrm{~d}_{i}[\mathrm{HS}]$ states increases with $i$. This is seen for $i=1,2,3$ in figure 4.
(5) A splitting of $5 \mathrm{~d}_{i}[\mathrm{HS}]$ in $\mathrm{Tb}^{3+}$ was never observed, indicating that the effect of $H_{\mathrm{f}}^{\text {so }}$ and $H_{\mathrm{d}}^{\mathrm{so}}$ is negligible in compounds.
(6) The shift model applies particularly well for $\mathrm{Tb}^{3+}$. It provides an easy method for assigning the bands observed.

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